



Social Choice and Individual Values: A Lockean Inquiry

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Abstract

Social Choice and Individual Values is a classic monograph by Kenneth Arrow (1921–2017), published by the National Bureau of Economic Research in 1951. In this monograph, Arrow argued that "rational" social choice is impossible under "capitalist democracy." Rationality requires transitivity in preference ordering, but this breaks down when voters are permitted to make pairwise comparisons among more than two options and two voters. In more specific terms, Arrow argues that a Social Welfare Function - determined through aggregating individual preferences - cannot satisfy five "natural conditions:" (i) Unrestricted Domain, (ii) Pareto Property, (iii) Independence of Irrelevant Alternatives, (iv) Nondictatorship, and (v) Transitivity. Therefore, rational social choice is impossible under a "capitalist democracy." Arrow's authority has been disputed in this paper. First, the so-called "voting paradox" does not exist in Arrow's model. Second, the causal relations established by artificial concepts cannot explain matters of fact. Finally, the transitivity rule breaks down in matters-of-fact reasoning. Arrow did not explain why this assumption is necessary for the voting system to be democratic.

Keywords: *Kenneth Arrow; Social Choice Theory; John Locke; Empiricism; JTB*

JEL Classification: B41, C0, P1.

1 Introduction

The primary objective of this paper is to assess the intellectual and practical values of the Social Choice Theory (SCT), a branch of welfare economics that developed in the second half of the last century. It is a formal analysis of collective decision-making rules in a "capitalist democracy," as articulated by Arrow (1951). Accordingly, Arrow's classic will be dissected to evaluate SCT's academic merit.

The SCT is not a single theory. It is a cluster of models and results that concern the aggregation of individual inputs (e.g., votes, preferences, judgments, welfare) into collective outputs (e.g., collective decisions, preferences, judgments, welfare).

This SCT formulation appears problematic because it contradicts the "Theory of Social Contract (TSC)" of John Locke and Rousseau, a foundational pillar of the democratic political system. According to this Theory, the state arises from a voluntary contract among free individuals, assumed to be equal in voting rights. This contract entails the "total submission" of all private possessions to the "Civil Society." The modern name of this sovereign power is the state or a nation. Finally, the sovereign power of the state is vested in the institution of government, to be administered by members of the state elected through a general franchise for a specific term.

The government, so established, acquires several features that demand critical attention to understand how the power is 'supposed' to be exercised under democratic governance. People (voters) own the government, meaning its sole purpose is to rule the nation, in accordance with the fundamental stipulations of the TSC. Since some citizens are elected to govern, individual members of the state have no right to disobey laws enacted legitimately. The notions of individual preference and choice are alien to the concept of democratic government. Here, the authority of social choice flows from the aggregate to the individual units, not the other way.

However, in the case of the SCT, the status of "social choice" is determined by aggregating personal choices. This is because there is no central authority within the group under consideration that can solicit members' opinions on policy decisions. Both entry and exit from the group are voluntary, which means that everything in this system is private; nothing is public. In the case of the social contract, entry is voluntary, but exit is not. No one can remain a member of the community by refusing to obey the majority's decision.

This also means that each member's vote is final and binding, implying that the voter will have no opportunity to record any other choice on the ballot paper. The voting pattern is transitive. This eliminates all uncertainties in calculating the final score. The question then arises as to the necessity of Arrow's imposition of other choices in determining the final result. Is it a necessary condition for voting in a "capitalistic democracy?"

The next issue in the Arrow paradox concerns the types of questions the SCT deals with:

Central questions are: How can a group of individuals choose a winning outcome (e.g., policy, electoral candidate) from a given set of options? What are the properties of different voting systems? When is a voting system democratic? How can a collective (e.g., electorate, legislature, collegial court, expert panel, or committee) arrive at coherent collective preferences or judgments on some issues, based on its members' individual preferences or judgments? How can we rank different social alternatives in order of social welfare? Social choice theorists study these questions not just by looking at examples, but by developing general models and proving theorems (List, 2022).

A closer look at the above paragraph should convince us that Arrow's SCT is a type of Aristotle's Delphic Knife because it can be used to address any problem related to group decision-making. Building a perfectly "rational" social choice theory is indeed possible by using the ideas represented by the artificial objects to be discussed later. Arrow has done that with magical mathematical elegance. This is a masterwork for economists who find pleasure in interpreting economic theories in mathematical

language. However, the fact that troubles ordinary minds is the utility of this sophisticated theory in addressing the Herculean problems that human beings worldwide are grappling with.

Second, examine the menu of topics provided by List (2022) that the SCT addresses. The characteristics of these are all different, which is why they have been given different names. How can we then apply the Delphic Knife and SCT to solve all these problems?

All these difficulties suggest that we have a professional obligation to review the SCT in light of the current world, which has already passed the quarter-century mark. The review will be futile if conducted according to the prevailing approaches that have failed to resolve the controversy. Accordingly, this paper will review Arrow's impossibility theorem by applying the empirical principles developed by John Locke. David Hume will come naturally in this discussion because, as Russell (1945) says, he brought Locke's theory to a dead end.

The paper is organized as follows. The next section summarizes the fundamental messages of Locke's "An Essay Concerning Human Understanding." Section 3 summarizes the relevant portions of Arrow's book, followed by an assessment of the SCT's abstract and obtuse nature in Section 4. The paper concludes in Section 5.

2 Human Understanding, Empiricism, and the Theory of Justified True Belief

Our interest is to examine the academic virtue of Arrow's SCT theory, which has influenced several branches of economics, particularly Welfare Economics. Like all other sciences, the knowledge of economics is the output of human understanding, which employs two broad approaches: rationalism and empiricism. Since social science knowledge is, by nature, inductive, empiricism is rightly considered the primary approach. The concept of JTB was derived from this approach. A satisfactory inquiry into the background and history of this theory would require in-depth research into Locke and Hume's contributions to epistemology. This in-depth survey cannot be adequately accommodated in a single article. Therefore, the following two subsections first describe Locke's contribution and then detail the meaning and role of JTB in producing knowledge in the social sciences, particularly economics.

2.1 John Locke on the Origin and Communication of Ideas

Locke lived in the 17th century, a period marked by the Enlightenment, also known as the Era of Reason. The question that sparked Locke's interest in pure philosophy, known as epistemology, is to understand the nature, origin, and limits of human knowledge. The answer given by the rationalist philosophers could not convince him:

It is an established opinion amongst some men that there are in the understanding certain innate principles; some primary notions, characters, as it were, stamped upon the mind of man, which the soul receives in its very first being; and brings into the world with it (Locke, 1690).

Locke rejected this rationalist theory, positing that individuals are born without innate mental contents, meaning all ideas originate from experience. He committed himself to working out this epistemological issue in 1670, which eventually culminated in "An Essay Concerning Human Understanding" two decades later. Human understanding is a faculty of the mind that investigates how we know what we know, its origins, and its limits. To articulate his theory of inductive knowledge, he appealed to a universal truth: our mind is a "blank slate" at birth. A question that naturally arises concerns its state at maturity, which Descartes' maxim may indicate: "I think; therefore, I am (Burns, 2001)."

In Book II, Locke posed the following question:

Let us then suppose the mind to be, as we say, white paper, void of all characters, without any ideas: How comes it to be furnished? Whence comes it by that vast store which the busy and boundless fancy of man has painted on it with an almost endless variety? Whence do they derive all the materials of reason and knowledge? To this, I answer, in one word, from EXPERIENCE. In that all our knowledge is founded, from that it ultimately derives itself. Our observation, whether about external sensible objects or about the internal operations of our minds perceived and reflected on by ourselves, is that which supplies our understandings with all the materials of thinking. These two are the fountains of knowledge, from whence all the ideas we have, or can naturally have, do spring (Locke 1690).

Locke calls these primary sources sensations and reflections. First, our senses convey distinct perceptions of things to our mind, caused by external factors or internal organs in various ways. For example, we feel the heat when our body comes into contact with fire, or our tongue feels sour when we eat bitter gourds. Similarly, we feel hungry when we go without eating for a while.

In other words, the sources of all our ideas are the effects internal and external objects produce on our body and mind, which Locke names sensations. The operation of the mind is the other source of ideas. Our brain is a vast repository of ideas derived from sensory experience. The human mind is never idle while awake. It reflects the ideas of sensation and generates a whole new set of ideas, such as thinking, doubting, believing, reasoning, knowing, and willing, that were never experienced by our body or soul.

This source of ideas every man has wholly in himself, and though it be not sense, as having nothing to do with external objects, yet it is very like it, and might adequately enough be called internal sense. However, as I call the other SENSATION, so I call this REFLECTION, the ideas it affords being such only as the mind gets by reflecting on its own operations within itself (Locke 1690).

Locke then divides all ideas present to the mind into two categories—simple and complex. Ideas, which enter the brain in unmixed manners, are called simple. Simple ideas, the building blocks of all human knowledge, are "suggested and furnished" to the mind by the methods mentioned above. Once the understanding has access to these ideas, it can repeat, compare, and unite them in an almost infinite variety. However, it cannot either create or destroy them. The most important quality of simple ideas is that the mind is entirely passive in receiving them while fully active in creating complex ideas. It performs this process in three ways: (i) combining several simple ideas into a compound one, i.e., creating complex ideas; (ii) setting two ideas, simple and/or complex, side by side without uniting them in one, giving the mind ideas of relations; and (iii) separating similar ideas of real existence from all others. This is called abstraction, the process by which all general ideas are created.

In Book III, "Of Words," Locke describes how the complex and straightforward ideas developed in our minds are communicated in public. In this book, Locke demonstrates that human knowledge is produced in two distinct phases. The first phase involves the mind's operations in conceiving complex ideas, which this paper refers to as "private knowledge." The second phase involves communicating these complex ideas to the public, which is defined as "public knowledge."

The vast literature on epistemology does not discuss these two distinctions in the context of human knowledge production. The importance of recognizing the existence of two phases of knowledge production lies in the fact that disagreements ordinarily arise when our understanding substantially differs from what the author(s) intended us to understand. When we agree with the meaning of a word or group

of words, we can never differ for long because the human mind thinks alike. Without making this premise, human communication is impossible.

Words are articulated sounds that humans make to communicate their ideas. This is a highly critical feature of human communication, as it can lead to confusion and controversy when the parties involved do not share the same understanding of the ideas conveyed by the words. In this regard, general and abstract words play critical roles in human understanding.

All things in the universe are distinct, suggesting they must have distinct names to distinguish one from another. Proper nouns, which signify a single entity like Adam and America, have been invented to represent these names. However, communications among human beings, ordinary or intellectual, would be virtually impossible with these kinds of words. Thus, most words used in human communication are general words or terms:

Words become general by being made the signs of general ideas, and ideas become general by separating from them the circumstances of time and place and any other ideas that may determine them to this or that particular existence. By this way of abstraction, they are made capable of representing more individuals than one, each of which has in it a conformity to that abstract idea, is (as we call it) of that sort (Locke 1690).

For example, "men and women" are general words, but "Adam and Eve" are specific words. Thus, "men and women" are abstract words that represent abstract ideas, while "Adam and Eve" are concrete words that represent concrete ideas. Abstract words refer to ideas or concepts with no physical referents, while concrete words signify objects or events accessible to our senses. Abstract ideas are the essence of genus and species, resulting from the faculty of human understanding's skillfulness.

Book IV is titled, "Of Knowledge and Opinion." The preceding three books explain how and from where the human mind obtains raw materials for understanding the relations among complex ideas and processing them into knowledge using the principles of logic. In this final book, he completes his mission by defining knowledge and distinguishing its key features. The book begins with this statement:

Since the mind, in all its thoughts and reasonings, hath no other immediate object but its ideas, which it alone does or can contemplate, it is evident that our knowledge is only conversant about them. Knowledge, then, is nothing but the perception of the connection of an agreement or disagreement and repugnancy of any of our ideas. In this alone, it consists. Where this perception is, there is knowledge, and where it is not, there, though we may fancy, guess, or believe, yet we always come short of knowledge. For when we know that white is not black, what else do we but perceive that these two ideas do not agree? When we possess ourselves with the utmost security of the demonstration, that the three angles of a triangle are equal to two right ones, what do we more but perceive that equality to two right ones does necessarily agree to, and is inseparable from, the three angles of a triangle (Locke, 1690).

We will now summarize Locke's theory of human understanding concerning the experienced ideas or matters of fact. Ideas for creating knowledge in the social sciences are primarily experienced in their raw form, called "simple ideas." The mind permutes and combines them in various ways, resulting in complex ideas. Human knowledge, which consists of complete statements or propositions, is a systematic arrangement of these ideas. This part of knowledge is private, but it is made public through the use of words. Finally, knowledge refers to agreement or disagreement with the ideas implied by words.

These agreements and disagreements are directly connected with the types of ideas under discussion. In scientific research, there are two types of ideas representing two types of objects: matters of fact and artificial concepts (Elahi, 2025). The "matter of fact" is a statement concerning things that are known through experience and observation. Therefore, they are subject to empirical testing and verification to generate knowledge.

On the other hand, artificial concepts refer to objects or ideas that do not exist in nature, meaning they are products of human intelligence and are found in logic, mathematics, and computing sciences. In this case, all concepts are defined precisely, indicating that the relations established among them are intuitively or demonstrably certain. In other words, knowledge concerning matters of fact is a matter of belief, but the knowledge concerning artificial objects is satisfactorily specific.

Here lies the connection between Locke's theory of human understanding and the modern theory of "propositional knowledge," commonly referred to as "Justified True Belief (JTB)." SCT is undoubtedly a "matter-of-fact" proposition, meaning it is a "matter of belief" and hence subject to empirical testing and verifications to determine its JTB nature. However, Arrow developed the impossibility theorem using artificial concepts whose characteristics are entirely different from those of the matter-of-fact.

The following pages aim to demonstrate that the unresolved and ongoing controversy on the SCT is primarily due to these epistemic properties of the two kinds of objects of human inquiry.

2.2 The Meaning of Justified True Belief (JTB) in Scientific Research

The subject we are discussing belongs to a very complex area of philosophy: epistemology. Accordingly, the subject is briefly described below to help readers understand the main thread of arguments in this paper.

Epistemology comes from two Greek words: episteme, meaning knowledge or understanding, and logos, meaning argument or reason. Accordingly, it is defined as a branch of philosophy that studies the nature, origin, and limits of human knowledge, along with distinguishing this vital information from mere 'opinions' (Locke, 1690).

Human knowledge is the output of our cognitive activities, the mental processes the faculty of human understanding conducts when we think and reason. In performing this activity, the mind utilizes 'ideas' as its raw material.

Modern epistemology is divided into two principal schools: rationalism and empiricism. Rationalists, such as René Descartes, Baruch Spinoza, and Gottfried Leibniz, believe that the fundamental source of knowledge is innate ideas (koinai ennoiai) that are naturally implanted in the human mind. They hold that reason is the chief source and test of knowledge, which is conducted through these ideas (Blanshard, 2025). Reality itself has an inherently logical structure, meaning a class of truths exists that the intellect can grasp directly. Accordingly, reasoning is deductive, i.e., a priori.

Empiricism has two major definitions (Fumerton, Quinton, & Duignan, 2025): First, it is a philosophical theory that posits all concepts originate from experience or are about or applicable to things that can be experienced. This theory of knowledge, also known as the theory of justification, was proposed by Sir Francis Bacon and further developed by John Locke and David Hume. Second, all concepts are divided into two categories: a posteriori and a priori. While the former can be applied independently of experience, the latter cannot. Accordingly, beliefs or propositions concerning matters of fact must be a posteriori, i.e., the trustworthy source of knowledge in science is experience.

The two approaches are conceptually inseparable because they use the same raw materials in their thinking and reasoning: ideas. Thus, the fundamental difference between these two definitions is the source of ideas, i.e., the extent to which the inquiry is dependent upon experience to produce its output (Markie & Folescu, 2023).

The key point to note here is that human knowledge, academia's primary purpose, is created in two distinct steps. The first step is entirely personal and private, involving principles from both rationalism and empiricism.

Knowledge, in everyday usage, has many meanings, one of which is "psychological conviction," which implies that something is known or understood with certainty. In philosophy, however, the term is treated differently; it is understood in the *factive* sense, meaning one cannot know anything unless it is the case. In this category, epistemology concentrates on "propositional" knowledge (Truncellito, 2007).

Propositional knowledge thus has three components- belief, truth and justification. First and foremost, knowledge is a mental state, meaning it exists in the mind of the beholder. Accordingly, to have propositional knowledge, the beholder must "believe" in the suggestion or thought processed by the mind.

Second, belief is a necessary condition for creating knowledge, but not sufficient to convert it into foolproof truth. This is because our beliefs could be false. Philosophy treats only "factive beliefs" as propositional knowledge, because only the truths of factive beliefs can be ascertained with evidence. This means our beliefs, at least in a philosophical sense, must relate to matters of fact.

Finally, our factive beliefs must be justified, because we might come up with factive true beliefs just by lucky guesses. This raises one of the most important and interesting questions in epistemology: What is the "rationally right" way to arrive at true belief? A general answer to this question is that beliefs must be justified with sound reasoning and solid evidence to be accepted as knowledge. Thus, propositional knowledge in epistemology is factive true belief derived through sound reasoning and solid evidence. This is the reason why propositional knowledge is referred to as Justified True Belief (JTB).

3 Social Choice and Individual Values: A Non-Mathematical Narrative

In the acknowledgement of the First Edition, Arrow says that he initiated the study in 1948 at the RAND Corporation, which conducts research under contract with the United States Air Force, and that he wrote the current monograph for the Corporation by June 1949. The RAND published the document as Monograph No. 12 in 1951. The same year, he also published it as a book by Yale University Press. By the time he published the journal version in the Journal of Political Economy in 1950.

As to the testimony of its impact on social science generally and economics specifically, we will present two witnesses. First, we quote Amartya Sen (1985), who compares Arrow's contribution with those of David Hume (Treatise of Human Understanding) and Bertrand Russell (Principia Mathematica), both of which were not immediately recognized by the intellectual community upon publication. However, Arrow's "impossibility theorem" was an instant classic:

Welfare economists, political theorists, moral philosophers, and others had to take note of what seemed like – and was indeed – a devastating and far-reaching result. Welfare economics, in particular, underwent a significant transformation. Responses took various forms, such as attempted refutations, proposed solutions, suggested compromises, defeatist resignations, and assertions that Arrow's analysis did not apply to this problem or that [...]. Within a comparatively

short time, the new subject of social choice theory was firmly established as a discipline with immediate and extensive implications for economics, philosophy, politics, and other social sciences. The literature on social choice theory has since expanded at a rate that has frequently alarmed journal editors, and it is now formidably large. The instant classic has become a lasting leader (Sen, 1985, pp. 1764-1765).

In his foreword to the third edition of the 1951 classic, Eric Maskin (2012) wrote:

With its publication in 1951, *Social Choice and Individual Values* initiated the modern theory of social choice, which examines how a society should choose among its various options based on the preferences of its individual members. There had been sporadic literature on the subject before Arrow, going back (at least) to Jean-Charles Borda and the Marquis de Condorcet in the late eighteenth century. However, the earlier essays lacked the generality and power of Arrow's approach, and the subject did not truly take off until the publication of *Social Choice*. However, it did take off: by the time the second edition was published, in 1963, there were already several hundred works building on the book. A recent search on Google Scholar yielded over ten thousand citations.

Against this background, the paper aims to offer insights that will pique the curiosity of our learned readers. The challenge begins with summarizing the book's relevant thematic ideas that constitute the SCT.

3.1 Meaning of Social Choice

The book is concise, comprising only 74 pages, divided into seven chapters. We will restrict ourselves only to Chapters I and II, where Arrow presents the research problem involved in collective decision-making.

A critical inquiry must begin with an understanding of the title's meaning and message. In our case, the title is "Social Choice and Individual Values." The last part of the title is not difficult to understand, but the first part may not be immediately apparent from the first reading of the paper. We can easily relate individual choice and values. Can we apply the same principle to social choice and individual values? We cannot, because the meanings of 'choice' and 'value' are largely sombre. On the other hand, the meaning of social choice is all but ambiguous.

A group is formed around an institution or organization. The state, an independent territory, is a complex network of various institutions, beginning with families. Individuals in each group have specific values that may not apply to others in the group. For example, individuals who value family life can never expect the government to support them in this regard. This is why family and government are identified as separate institutions, although from a mathematical perspective, they are simply groups.

Thus, the idea implied by "family" does not align with that of "government," even though both are, by definition, social groups. From Locke's perspective, no propositional knowledge can be created by associating "social choice" with "individual values." This is the first point we want to note that Arrow's intellectually acclaimed book is epistemologically questionable. Then, this highly complex mathematical book is virtually illegible to ordinary readers and policymakers, resulting in little policy impact.

3.2 Nature of Social Choice Paradox in Arrow (1951)

Chapter I, Introduction, begins with this heading: "The Types of Social Choice." Here are the first two paragraphs of Chapter I:

In a capitalist democracy, there are essentially two methods by which social choices can be made: voting, typically used to make "political" decisions, and the market mechanism, typically used to make "economic" decisions. In emerging democracies with mixed economic systems, such as Great Britain, France, and Scandinavia, the same two modes of making social choices prevail, although more scope is given to voting and to decisions based directly or indirectly on it, and less to the rule of the price mechanism. Elsewhere in the world, and even the smaller social units within the democracies, social decisions are sometimes made by single individuals or small groups and sometimes (more and more rarely in this modern world) by a widely encompassing set of traditional rules for making the social choice in any given situation, e.g., a religious code.

The last two methods of social choice, dictatorship and convention, have a certain definiteness in their formal structure that is absent from voting or the market mechanism. In an ideal dictatorship, there is but one will involved in choice; in an ideal society ruled by convention, there is but the divine will or, perhaps, by assumption, a common will of all individuals concerning social decisions, so in either case, no conflict of individual wills is involved. Voting and the market, on the other hand, are mechanisms for aggregating the preferences of many individuals to form social choices. The methods of dictatorship and convention are, or can be, rational in the sense that any individual can be rational in their choices. Can such consistency be attributed to collective modes of choice, where the wills of many people are involved (Arrow, 2012, pp. 1-2)?

These two paragraphs describe Arrow's research problem in the book. By "capitalist democracy," Arrow refers to the United States of America. In plain English, rational social choice-making is possible under dictatorship and convention, but not under capitalist democracy. This is because social choice-making under dictatorship and conventions does not require aggregating individual values, whereas it does under capitalist democracy.

Arrow explains this problem by citing the well-known "voting paradox." A community consisting of three individuals is challenged to make a social choice among three alternative modes of social action (e.g., disarmament, cold war, or hot war), as an individual consumer makes a consumption decision under conditions of constant wants and variable price-income situations in the ordinary ordinal utility analysis. The "rational behaviour" of the community would imply ordering the three alternatives according to its collective preferences once and for all, and then, in any given case, choosing the alternative preferred by the majority of community members. Arrow uses three numbers, 1, 2, and 3, to represent the three individual members of the community, and three letters, A, B, and C, to stand for the three alternative social states or policy actions. He then assigns or imposes the following preference orderings for the three individuals to examine the nature of social choices that emanate if the principles of majority rule are upheld.

Individual 1: prefers A to B and B to C; therefore, A to C by transitivity

Individual 2: prefers B to C and C to A; therefore, B to A by transitivity

Individual 3: prefers C to A and A to B; therefore, C to B by transitivity.

In plain English, the preference orderings of the three individuals stated above may be described as follows: Individual 1 prefers A to B and then B to C. By transitivity, this means individual 1 also prefers A to C. We can describe the preference orderings of individuals 2 and 3 in the same manner. It is also evident that, given these preference orderings, there is no social choice that complies with the

majority rule. This is true when we apply the transitivity restriction, imposing the pairwise comparisons required by the first property in Arrow's model. The three individuals have chosen three different policy options: Individual 1 prefers option A, Individual 2 prefers option B, and Individual 3 prefers option C.

Let us count pairwise policy choices, ignoring transitivity as Arrow did. In other words, choices deduced for individuals 1, 2 and 3 by applying the rule of transitivity are excluded from the matrix of two-alternative voting preferences. Moreover, there is no social choice in this scenario either, because there are no clear-cut majority preferences: individuals 1 & 2 prefer B to C; individuals 2 & 3 prefer C to A and individuals 1 & 3 prefer A to B. In other words, two individuals prefer B to C; two prefer C to A and two prefer A to B. The social scenario described above fails to satisfy the condition of rationality as we ordinarily understand it.

Arrow now raises a second question: "Can we find other methods of aggregating individual tastes which imply rational behaviour on the part of the community and which will be satisfactory in other ways (Arrow, 1950, p. 329)?"

To answer this question, he directs our attention to the problems associated with making choices from the notion of a social welfare function, as developed by Bergson (1938), Lange (1942), and Samuelson (1947). The Bergson-Samuelson social welfare function is derived by aggregating individual choices, as represented by indifference curves, where consumers maximize utility subject to a budget constraint. This derivation, in turn, suggests that society must maximize utility when making choices for all its members, which occurs when the choice reflects the preferences of the majority. The problem with this approach is that the measure of utility is ordinal rather than cardinal, which precludes interpersonal comparisons. Besides voting, several compensation principles (Kaldor, 1939; Scitovsky, 1942; Hicks, 1939a) have been proposed to address this conceptual issue. However, these compensation principles are as problematic as majority voting, because the aggregated "individual preferences may lead to a pattern of social choice which is not a linear ordering of the social alternatives."

Arrow believes that the difficulties involved in making social choices described above are *general*, and his 1950 paper was composed to affirm this conviction. The rest of his paper is organized under four sections: Section II, titled 'Definitions and Notation', elaborates his notion of 'social states' and uses the language of set theory and symbolic logic to state this general problem of social choice. This involves replacing actual social agents that make preferences and actual social states with axioms, definitions, and lemmas. Section III - titled 'The Social Welfare Function' - completes his model by directly linking this general model with Bergson-Samuelson's formulation intended to make social welfare judgments. Here, Arrow underlines positive associations between social and individual values and imposes three additional restrictions – (i) irrelevance of independent alternatives, (ii) condition of citizens' sovereignty and (iii) non-dictatorship condition. Arrow derives his impossibility theorem in Section IV, titled 'The Possibility Theorem for Social Welfare Functions'. In contrast, Section V, titled 'Some Implications for the Formulation of Social Welfare Judgments', concludes the paper.

At the end of Section IV, Arrow summarizes his opinion about making social choices under majority rule:

If there are at least three alternatives among which the members of the society are free to order in any way, then every social function satisfying conditions and a social ordering satisfying Axioms I and II must be either imposed or dictatorial. The Possibility Theorem demonstrates that, if no prior assumptions are made about the nature of individual orderings, no method of voting will eliminate the paradox of voting discussed in Part I, whether it is plurality voting or any scheme of

proportional representation, regardless of its complexity. Similarly, the market mechanism does not create a rational social choice (Arrow, 1950, p. 342).

He restates his theorem as follows (the *Italic* is original) in the following section: *If we exclude the possibility of interpersonal comparisons of utility, then the only methods of passing from individual tastes to social preferences which will be satisfactory and which will be defined for a wide range of sets of individual orderings are either imposed or dictatorial* (Arrow, 1950, p. 342).

In 2014, Arrow summarizes his impossibility theorem in the same way he articulated it about six and a half decades ago: A society, Arrow says, "must make a choice binding on all its members." On a particular occasion, this society must make a social choice from a set of S alternatives. The size of society and the number of alternatives contained in S are all assumed to be finite. It is also assumed that all members of society, represented by the i th individual, have a preference ordering R_i over all conceivable alternatives, which is transitive and complete. The objective of the model is to "seek a social preference ordering, R , from all alternatives from which the social choices for any given set of alternatives, S , can be derived as the maximal elements of S under the ordering R (Arrow, 2014, p. 144)."

The social ordering model R , in turn, must satisfy two properties. First, R must reflect the preference orderings of every individual member of the society made by using information relevant to the given preference set S . "Further, it should be defined for any conceivable set of individual alternatives, i.e., it is a functional, called the social welfare functional, from the vectors of individual preference orderings to preference ordering (Arrow, 2014, p. 144)." The second property, termed *irrelevance of independent alternatives*, states that 'the choice from any two-alternative set depends only on the preferences of individuals as between those alternatives. The first property has been stated in different ways, one of which is the weak Pareto Principle: "if everyone strictly prefers x to y , then x is preferred to y in social ordering. The impossibility theorem then states that there is no social welfare functional with the required properties (Arrow, 2014, p. 144)."

In real-life situations, Arrow insists, an election is a clear example that illustrates the scope of the theorem. In any election, voters reveal their personal preferences by choosing the candidate they prefer. "The theorem states that if a candidate is chosen among three or more, he or she would not necessarily be chosen in a two-candidate race against any other candidate. If all voters and all candidates are treated alike, the voting method reduces to majority voting in a two-candidate race, and then the theorem states that among the three candidates, it is possible that A would get a majority against B , B against C , and yet C against A (Arrow, 2014, pp. 144-5)."

This last quotation then summarizes in a nutshell the fundamental principles of Arrow's social choice theory, built under two big 'ifs': First, if a candidate (a social state) chosen in a three- or more-contestant race would not necessarily be chosen in a two-candidate race. Second, *if* all voters and candidates are assumed to be alike, the competition reduces to a two-candidate race, making it impossible to find a majority supporting a single candidate.

The above summarization of Arrow's social choice ideas in plain English has perhaps made his theory accessible and understandable to ordinary individuals, albeit at the cost of losing some of the paper's intellectual illumination. What is nevertheless important is to examine how this theory, which is essentially based on speculative assumptions, is applied to the rationality of social science investigations. The reason the foundation of theory is described as speculative is that the real-life social institutions and issues have little bearing on the 'relations' portrayed in the model. Moreover, when our mind begins reasoning to examine these relations by using the symbols and signs assigned by Arrow, we can hardly think of anything else but what he wants us to see or believe.

4. Some Critical Observations on "Social Choice and Individual Values"

Sections 3 and 4, respectively, attempted to summarize the theme ideas of Locke's empirical philosophy and Arrow's SCT. This section aims to evaluate the academic merits of Arrow's ideas in the context of Locke's theory of human understanding. This exercise, not attempted before, seems critical because Locke taught us which type of theory or doctrine can be considered "propositional knowledge," and Arrow claims to provide a theory of social choice that needs to be validated by Locke's empirical principles.

4.1 Ideas: Theorem vs. Theory

Ideas are the raw materials used in the reasoning process, which eventually create human knowledge. As already discussed, the fundamental difference between rationalists and empiricists concerns the primary role these ideas play in creating knowledge in different areas of human affairs. Rationalists rightly emphasize the importance of innate ideas in the creation of metaphysical knowledge. However, the creation of scientific knowledge, which involves ideas obtained from experience, is fundamentally more critical. Besides these two types of ideas, there is a third type that mathematical philosophers, such as Pythagoras, first developed. These ideas were key instruments in ancient metaphysical thinking. In modern times, they have assumed enormous importance in scientific research.

To distinguish these mental images from others, Hume (1748) defined them simply as "ideas," while the rest were identified as "matters of fact." He divided all objects of human reasoning into two kinds: relations of ideas and relations of matters of fact. Hume's classification is somewhat confusing because he has defined all objects of human inquiry as "ideas." To overcome this difficulty, we will refer to them simply as "artificial concepts" because they lack referents.

This difference reflects the fundamental distinction between the words "theorem" and "theory." A theorem is a single, specific statement that has been rigorously proven true within a formal system, like mathematics. In contrast, a theory is a broader, overarching framework or explanation that is supported by evidence but not definitively proven, and can be revised as new information emerges. For example, if the angle opposite the hypotenuse is 90 degrees, the triangle is called a right-angled triangle. The relation between the angles of a triangle was discovered by the ancient Greek philosopher Pythagoras, for which it is known as the Pythagorean theorem. This kind of confidence cannot be expected in the case of theories representing matters of fact. All economic theories, particularly of a microeconomic nature, fall within this category.

Difficulties with social choice theory, a branch of welfare economics, arise from this point. For Arrow's conclusion in "Social Choice and Individual Values" is a theorem, not a theory; it is known as "Arrow's Impossibility Theorem." Some Arrow admirers might dismiss this difference as mere semantics. Before they do that, they should be careful about its epistemological implications.

Arrow examines the problems of social choice in a "capitalistic democracy," in which collective decisions concerning political issues are made through voting. In contrast, those related to economic issues are determined through the market mechanism. This means that the same person participates in both the election and the market transaction as a citizen, sharing the state's sovereign authority, and as a property owner, commanding purchasing power. Since voting and buying are distinct activities, individuals must have different motives for voluntarily participating in them. These motives are expected to vary from one individual to another. There is no social choice problem if all individuals behave identically.

Arrow (1951) represents real people with the numbers 1, 2, and 3, which lack behavioural properties. They are just intellectual instruments with no AI; their behaviours are always determined by their users. This means Arrow chose the preference values so that the matrix creates a "voting paradox." This cannot be a model of social science that studies human behaviour in various affairs. In other words, Arrow's SCT, articulated in his classic "Social Choice and Individual Values," does seem to qualify as a social science model.

4.2. Is there really a Social Choice Paradox in "Capitalistic Democracy?"

This is a fascinating question about Arrow's impossibility theorem. As explained above, the term "theorem" refers to the relationship between artificial concepts. On the other hand, a "theory" explains the cause-and-effect relations among matters of fact. While social scientists can use artificial concepts to clarify their theories, experts in artificial concepts cannot do the same; that is, they cannot explain social theories in their artificial language. This explanation follows directly from Hume's theory of human understanding.

We put forward this point to argue that Arrow has contradicted himself. As an American citizen, he did not highlight the flagrant weaknesses of the American voting system, which are readily apparent. Instead, he built a mathematical puzzle around his "ideal" system of "rational" social choice theory, which means very little to ordinary economists, as they do not understand it. However, expert mathematicians interested in experimenting with their skills in economic issues accepted his mathematical magic work as a paradigm-shifting idea that eventually helped him win a Nobel Prize.

Several key points can be noted here to highlight the methodological foundation of Arrow's dissertation. First, Arrow did not develop a new methodological approach; he draws on the longstanding "voting paradox," credited to the 18th-century French scholar Nicolas de Condorcet. As noted above, Condorcet was a versatile scholar who turned his curiosity to examine problems of group decision-making under free and equal membership. Due to his political background, this mathematical puzzle eventually became known as a paradox in democratic elections. Although it is clear that Condorcet was interested in the mathematical properties, our survey does not indicate that he was concerned with the appointment of the government's political leader through voting.

The originality in Arrow's approach lies in his application of Condorcet's idea exclusively to the election process conducted in a "capitalist democracy." By doing this, he has introduced "suspicion" into the merit of electoral systems practiced globally. Fortunately, this suspicion has not shaken public confidence in the virtues of these electoral systems. This is evident because a very insignificant proportion of economists and social scientists have adopted Arrow's thesis.

Third, the Delphic Knife Arrow has created a contradiction. A "group" is formed when several individuals voluntarily come together to achieve a common objective. Stated otherwise, the word "group" is a mathematical term that denotes an assembly or association formed for a particular purpose. This means that group members' behaviour varies significantly across groups. An individual can be a member of several groups, and they behave according to each group's purpose. Ignoring this fact is an intellectual crime that the masters of the economics discipline have been committing routinely.

All these points undermine the credibility of Arrow's impossibility thesis. However, they do not answer our query: Is there really a paradox in Arrow's dissertation?

Let us take a closer look at Arrow's voting paradox example.

Voter/Preference	1st	2nd	3rd
Voter 1	A	B	C
Voter 2	B	C	A
Voter 3	C	A	B

Arrow says, and we accept, that these are preference orderings of voters labelled 1, 2, and 3. However, these are numbers, not persons. If they were persons, then they would have names like Sham, Suson, and Sundra. If these three names substitute for the numbered voters, our minds would immediately want to know why they made those preference orderings. However, no such thoughts arise when we consider Arrow's example. Moreover, the numbers cannot order their preferences, which means that Arrow himself imposed those preferences.

The next question that arises is: Why did Arrow choose these preference orderings? He could have made the following preference orderings:

Voter/Preference	1st	2nd	3rd
Voter 1	A	B	C
Voter 2	A	B	C
Voter 3	A	B	C

In this example, there is no "voting paradox." A is the consensus candidate in this voting exercise. This means that Arrow has to justify why the example he has chosen is the only possible preference ordering in an experiment with three voters and three policy options.

Arrow did not explain, and our master social choice theorists did not consider it necessary to raise this question. However, we now know that David Hume provided the clue to this paradox in the first half of the 18th century. This unnecessary controversy would never have arisen in the social science literature if our master economists had been more professional, instead of chasing reputation and money.

Conclusion

Social Choice Theory (SCT) is a sub-discipline within the broader field of welfare economics. American mathematical economist Kenneth Arrow introduced this Theory in his abstract mathematical classic "Social Choice and Individual Values," first published as a monograph by the Cowles Foundation in 1951. The monograph has attracted scholars across political science, sociology, legal studies, philosophy, and, above all, economics. Maskin (2012) cites two reasons for this popularity. First, the abstract formulation of the social choice problem has made it widely applicable. SCT begins with a society and a set of social alternatives, which, depending on the context, could be almost anything.

Second, the SCT is founded on the theory of the social welfare function (SWF) developed by Bergson (1938). In this paper, Bergson describes the precise form of the value judgments required to derive the conditions of maximum economic welfare. According to Arrow, an SWF is any rule for determining society's preferences over the set of social alternatives based on the preferences of individual members. Put differently, the SWF is determined by aggregating individual preferences, which must satisfy five "natural conditions:" (i) Unrestricted Domain, (ii) Pareto Property, (iii) Independence of Irrelevant Alternatives, (iv) Nondictatorship, and (v) Transitivity.

Arrow argues that no SWF satisfies all five "natural conditions," meaning a social equilibrium that meets democracy's majority rule principle is "impossible" in a "capitalist democracy." If we accept this opinion, we should rename SCT as "social choice paradox," since theory and paradox are distinct concepts. Theory makes a proposition, which may be refuted through appropriate verification. On the contrary, a paradox is a contradiction, which is either true or false.

The analysis done in the preceding pages refutes Arrow's claims. First, it shows that there is no "voting paradox" in his SCT model. Second, the versatility and usefulness of the SCT in Arrow's model are not virtues. On the contrary, this property demonstrates that the model's theoretical and practical merits are highly contestable. Finally, the main objection against voting in a democracy is that the system is not "transitive" pairwise. This is true. The democratic system of governance cannot work if the votes cast are not final and binding. This raises the most interesting question for proponents and admirers of the social choice paradox: Why does Arrow insist on imposing a "pairwise" transitive vote-counting electoral system for a "capitalist democracy?" Arrow did not ask himself this question. Moreover, we do not know whether his proponents and admirers have done this.

Footnotes

One may criticize Arrow's social choice paradox and form an opinion about its intellectual virtues from the very first sentence which sets up his mathematical odyssey: "In a capitalist democracy, there are essentially two methods by which social choices can be made: voting, typically used to make 'political' decisions, and the market mechanism, typically used to make 'economic' decisions."

By a "capitalist democracy," Arrow means the United States of America, of which he is a citizen. Naturally, delving into the problems of social choice in America provided him with the fundamental inspiration to write this obtuse odyssey.

We should not have any difficulty in appreciating and recognizing his mathematical genius had he meant it as an advanced mathematical exercise in economics. Regrettably, Arrow presented it as a critique of democracy's voting system, which is believed to be the only political system capable of bringing about stability in public administration and good governance.

Both Arrow's admirers and adversaries are well aware of this truth. However, this truth does not bother their curiosity because Arrow avoided the everyday language of intellectual communication and presented his "propositions" in mathematical language whose "ideas" are artificial. The three candidates in his "voting paradox" example do not pique our curiosity about their personal, social, and political characteristics because they are represented by letters. If A, B and C are replaced by the words, democrat, republican, and independent, our faculty of understanding will be very curious to collect that data. Similarly, numbers 1, 2, and 3 are replaced by actual names whose personal, social and political data are known to us, our minds will closely inspect the voting preferences imposed by Arrow whenever they contradict the factual scenario. For example, a die-hard Republican voter will never prefer a Democrat candidate as their first choice and vice versa.

Second, social choice is defined as the study of how to combine individual preferences into a collective decision, such as a group choosing a candidate or policy. It analyses the relationship between individual values and group outcomes, examining the rules and mechanisms used for aggregation, such as voting systems. A central theme is determining whether it is possible to create a fair and consistent system for aggregating preferences without paradoxes, a problem famously explored by Kenneth Arrow's impossibility theorem (AI-generated).

This conception of social choice is confusing. First, to describe something as a social choice, there must be an association consisting of all eligible voters who cast their votes "voluntarily." Accordingly, an aggregated figure can never be a social choice because we do not know whether the votes were voluntary, i.e., free of coercion or bias. This is the "ideal" situation of voting in a democratic election, not an "actual" one. Therefore, it is a mistake to take an actual voting situation as an ideal one. Moreover, Arrow is found guilty of committing this crime.

The difficulty we are grappling with, Arrow's social choice paradox, may be explained by the following statement made by J. M. Keynes at Alfred Marshall's obituary. Here Keynes expounds why it is challenging to find paradigm-shifting authors in economics.

The study of economics does not seem to require any specialized gifts of an unusually high order. Is it not, intellectually regarded, a very easy subject compared with the higher branches of philosophy and pure science? Yet good, or even competent, economists are the rarest of birds. An easy subject, at which very few excel! The paradox may be explained, perhaps, by the fact that the master economist must possess a rare combination of gifts. He must meet a high standard in several areas and combine talents not often found together. He must be a mathematician, historian, statesman, philosopher—to some degree. He must understand symbols and speak in words. He must contemplate the particular in terms of the general and touch abstract and concrete in the same flight of thought. He must study the present in the light of the past for the purposes of the future. No part of man's nature or his institutions must lie entirely outside his regard. He must be purposeful and disinterested in a simultaneous mood, as aloof and incorruptible as an artist, yet sometimes as near the earth as a politician (DeLong, 2017).

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